

# Vacuum Dominance and Holography

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## Abstract

A cosmological event horizon develops in a vacuum-dominated Friedmann universe. The Schwarzschild radius of the vacuum energy within the horizon equals the horizon radius. Black hole thermodynamics and the holographic conjecture indicate a finite number of degrees of freedom within the horizon. The average energy per degree of freedom equals the energy of a massless quantum with wavelength of the horizon circumference. This suggests identifying the degrees of freedom with the presence or absence, in each Planck area on one horizon quadrant, of a  ${}_0S_2$  vibrational mode of the horizon with a  $z$  axis passing through that area. Pressure waves on the horizon (the superposition of  ${}_0S_2$  vibrational modes) can be envisioned to propagate into the observable universe within the horizon at the speed of light. So, the vacuum energy and pressure throughout the observable universe could (in principle) be determined from the vacuum equation of state.

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## A vacuum-dominated universe is asymptotic to de Sitter space

Friedmann's equation for the radius of curvature  $R$  of a closed homogeneous isotropic universe is

$$\dot{R}^2 - \left(\frac{8\pi G}{3}\right) \left[ \varepsilon_r \left(\frac{R_0}{R}\right)^4 + \varepsilon_m \left(\frac{R_0}{R}\right)^3 + \varepsilon_v \right] \left(\frac{R}{c}\right)^2 = -c^2$$

where  $\varepsilon_r$ ,  $\varepsilon_m$ ,  $\varepsilon_v$  and  $R_0$  are, respectively, the present values of the radiation, matter and vacuum energy densities and the radius of curvature.

Astrophysical measurements indicate the expansion of the universe is accelerating, and the energy density of the universe is dominated by a cosmological constant/vacuum energy density with  $\Omega_\Lambda = 0.7$ . The cosmological constant is related to the vacuum energy density by  $\Lambda = 8\pi G\varepsilon_v/c^4$ . At late times,  $R \rightarrow \infty$  and the radiation and matter energy density (and the curvature energy) are driven to zero by the expansion of the universe. The Friedmann equation becomes

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G\varepsilon_v}{3c^2} = \frac{\Lambda c^2}{3}.$$

So, our universe is asymptotic to a de Sitter space (the vacuum solution to Einstein's equations with a positive cosmological constant), and the asymptotic value of the Hubble constant is  $H = \dot{R}/R = c\sqrt{\Lambda/3}$ .

## Horizon radius equals Schwarzschild radius of vacuum energy within horizon

There is a cosmological event horizon in a de Sitter space, with horizon radius  $R_H = c/H = \sqrt{3/\Lambda}$  (see, e.g., [1, 2, 3]). Taking the Hubble constant as  $H_0 = 65 \text{ km sec}^{-1} \text{ Mpc}^{-1}$ , the critical density  $\rho_c = \frac{3H_0^2}{8\pi G} = 7.9 \times 10^{-30} \text{ g cm}^{-3}$ , the vacuum energy density  $\varepsilon_v = 0.7\rho_c c^2 = 5.0 \times 10^{-9} \text{ g cm}^2 \text{ sec}^{-2} \text{ cm}^{-3}$ , and  $\Lambda = 1.0 \times 10^{-56} \text{ cm}^{-2}$ .

The total vacuum energy within the horizon radius  $R_H$  is  $\varepsilon_v \frac{4\pi}{3} \left(\frac{3}{\Lambda}\right)^{3/2} = \frac{c^4}{2G} \sqrt{\frac{3}{\Lambda}}$ . The mass equivalent of this energy,  $M = \frac{c^2}{2G} \sqrt{\frac{3}{\Lambda}}$ , is the mass of the observable part of the vacuum-dominated universe (analogous to Wesson's "Einstein mass" [4]).

The Schwarzschild radius  $R_S$  of a mass  $M$  is  $R_S = 2GM/c^2$ , so the Schwarzschild radius of the vacuum energy within the horizon is  $R_S = \sqrt{3/\Lambda} = R_H$ . Therefore, the volume within the horizon radius  $R_H$  is analogous to the inside of a black hole with radius  $R_S = \sqrt{3/\Lambda} = R_H$ .

The relation  $\frac{M}{R} \approx \frac{c^2}{2G}$ , where  $M$  and  $R$  are the mass and radius of the universe, is sometimes obtained by invoking Mach's principle [5, 6]. Because the mass equivalent of the vacuum energy within the horizon is  $M = \frac{c^2}{2G} \sqrt{\frac{3}{\Lambda}}$ , the relation  $\frac{M}{R} \approx \frac{c^2}{2G}$  holds whenever

the radius of a vacuum-dominated Friedmann universe approaches or exceeds the horizon radius and is approximately true today.

## Finite degrees of freedom within the horizon

The area of the cosmological horizon at late times in a vacuum-dominated Friedmann universe is  $A = 4\pi R_H^2 = 12\pi/\Lambda$ . The holographic conjecture then indicates the number of observable degrees of freedom in the universe is  $N = A/4 = \pi R_H^2 = 3\pi/\Lambda$ , where  $A$  is measured in Planck units. Thus,  $N = \pi R_H^2/\delta^2 = 3\pi/(\Lambda\delta^2)$ , where  $\delta$  is the Planck length.

Black hole thermodynamics alone indicates the number of observable degrees of freedom in a black hole with surface area  $A$  is  $N = A/4$ , where  $A$  is measured in Planck units. So, considering black holes within the horizon with radii approaching the horizon radius, the above result from the holographic conjecture is not surprising,

## Average energy per degree of freedom

For *any* Schwarzschild black hole, with  $M = \frac{c^2 R_S}{2G}$ , the number of degrees of freedom is  $N = A/4 = \pi R_S^2/\delta^2$ . Defining the Planck length by  $\delta = \sqrt{\frac{\hbar G}{c^3}}$ , instead of the usual  $\delta = \sqrt{\frac{\hbar G}{c^3}}$ , the average energy per degree of freedom is  $\frac{E}{N} = \frac{c^4 \delta^2}{2\pi G R_S} = \frac{\hbar c}{2\pi R_S}$ .

Considering the volume of a vacuum-dominated universe within the horizon, the total vacuum energy within the horizon is  $\frac{c^4}{2G} \sqrt{\frac{3}{\Lambda}} = \frac{c^4 R_H}{2G}$ , and the number of degrees of freedom is  $N = \frac{3\pi}{\Lambda\delta^2} = \pi \frac{R_H^2}{\delta^2}$ . So, the average energy per degree of freedom in a vacuum-dominated Friedmann universe at late times is  $\frac{c^4 \delta^2}{2\pi G R_H} = \frac{\hbar c}{2\pi R_H} = \frac{\hbar c}{R_H}$ . Since  $\hbar c = 197.32$  MeV Fermi, the average energy per degree of freedom in the observable vacuum-dominated universe at late times is about  $10^{-33}$  electron volts. The mass equivalent of this energy is approximately the quantum of mass (about  $2 \times 10^{-65}$  g) identified by Wesson [4].

The energy of a massless quantum with wavelength  $\lambda$  is  $hc/\lambda$ . So, if the energy per degree of freedom in the observable vacuum-dominated universe at late times is carried by massless quanta, the average wavelength of those quanta is  $2\pi R_H = 2\pi \sqrt{3/\Lambda} = 1.09 \times 10^{29}$  cm, the circumference of the horizon. The average frequency of these quanta is  $c/\lambda = 2.75 \times 10^{-19}/\text{sec} = 8.67/10^{12}$  yr.

The surface temperature of a Schwarzschild black hole of mass  $M$  is  $T = \frac{\hbar c^3}{8\pi G M k} = \frac{\hbar c}{4\pi R_S k}$ . A Schwarzschild black hole with solar mass  $2 \times 10^{33}$  g has a surface temperature of  $6 \times 10^{-8}$  °K, so the surface of any large Schwarzschild black hole is a low temperature environment. In particular, the surface temperature of a Schwarzschild black hole with radius  $R_S = \sqrt{3/\Lambda} = R_H$  is  $T = \frac{\hbar c}{4\pi R_H k} = 10^{-30}$  °K.

## Possible connection to holography

The holographic conjecture and black hole thermodynamics suggests the number of bits of information necessary to describe the observable universe of radius  $R_H$  within the horizon is  $N = \pi \frac{R_H^2}{\delta^2}$ . This is the number of “pixels” of area  $\delta^2$  on the surface of one quadrant of the horizon.

The lowest frequency (lowest energy) vibrational mode of a spherical surface is the  ${}_0S_2$  “football” mode [7] where the sphere is alternately elongated and compressed along its  $z$  axis. The pressure waveform corresponding to this mode, on the circumference in any plane including the  $z$  axis, has wavelength  $\pi R$ , where  $R$  is the radius of the sphere. A massless quantum with wavelength  $\pi R_H$  has twice the energy of a massless quantum with wavelength  $2\pi R_H$  and thus twice the average energy per degree of freedom in the observable vacuum-dominated universe at late times. Because the surface temperature on the horizon is  $10^{-30}$  °K, only the lowest energy excitations should occur on the surface, if the excitations have integral spin.

Suppose each pixel (bit) on one quadrant of the horizon has value 1 if the  $z$  axis of a lowest frequency vibrational mode lies within that pixel and zero (no excitation) otherwise. If values 0 and 1 are randomly assigned to each pixel,  $N/2$  lowest frequency modes will be excited, and the average energy per degree of freedom will be  $\frac{hc}{2\pi R_H}$ . Because of the symmetry of the lowest frequency vibrational mode  ${}_0S_2$ , specifying the bits on one quadrant of the horizon determines the pressure waveform (superposition of  $N/2$  lowest frequency modes) on the entire horizon.

The pressure pattern on the horizon (the superposition of the  $N/2$  lowest frequency modes excited) can be envisioned to propagate into the universe within the horizon at the speed of light. Because the equation of state of the vacuum energy is known, a gigantic calculation (possible in principle) could determine the vacuum energy and pressure throughout the observable universe.

The scalar field responsible for the vacuum energy might be related to the size of compact extra dimensions, as suggested by Turner [8] and considered in [9]. If so, the vacuum energy and pressure at any point could be related to perturbations in the size of the compact dimensions at that point. Since distortions of compact dimensions are seen in string/M theory as fermions and bosons, wave patterns on the horizon might suffice to reconstruct the distribution of matter and energy within the observable universe. This approach could only be applied to Schwarzschild black holes within our universe if the equation of state for the energy inside those black holes could be determined. Although this discussion focuses on closed vacuum-dominated Friedmann universes, the argument can be extended to flat and open vacuum-dominated Friedmann universes.

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